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ABSTRACT

We consider a multi-class multi-mode variable demand network equilibrium model where the mode choice model is given by aggregate hierarchical logit structures and the destination choice is specified as a multi-proportional entropy type trip distribution model. The travel time of transit vehicles depends on the travel time of other vehicles using the road network. A variational inequality formulation captures all the model components in an integrated form. A solution algorithm, based on a Block Gauss-Seidel decomposition approach coupled with the method of successive averages results in an efficient algorithm which successively solves network equilibrium models with fixed demands and multi-dimensional trip distribution models. Numerical results obtained with an implementation of the model with the EMME/2 software package are presented based on data originating from the city of Santiago, Chile.

Keywords: network equilibrium, variable demand, hierarchical logit structure, simultaneous equilibrium.

RÉSUMÉ

Nous considérons un modèle d'équilibre de réseau multi-classes et multi-modes. Le modèle de choix modal est donné par une fonction logistique hiérarchique et le modèle de choix de destination est donné par un modèle entropique multi-proportionnel. Une formulation sous forme d'inégalité variationnelle intègre toutes les composantes du modèle. Un algorithme de solution, basé sur une décomposition de bloc Gauss-Seidel qui utilise la méthode des moyennes successives, résoud cette inégalité variationnelle efficacement. Des résultats numériques obtenus avec une implantation de ce modèle avec le progiciel EMME/2 pour la ville de Santiago, Chili, sont également présentés.

Mots clefs: équilibre de réseau, demande variable, modèle logistique hiérarchique, équilibre simultané.

1. Introduction

Over the past twenty years, the models used in transportation planning for congested urban networks received a lot of attention from researchers and practitioners. One of the principal issues studied and debated is that of the consistency of the models used for predicting the demand for travel and the network models used to determine the levels of service. One seeks to ensure that the levels of service which are used to compute the demand would be reproduced if that demand were used again to compute the levels of service. The keywords that we find in the literature regarding these models are "integrated", "combined", "simultaneous", "feedback" and "variable demand". The recent San Francisco Bay Area lawsuit (see Garrett and Wachs, 1996, p.199) shows that the importance of this issue has social and political impacts.

The model presented in this paper is inspired from the work carried out for the ESTRAUS project in Santiago, Chile (ESTRAUS, 1985-1998) and is a complex multi-class multi-mode variable demand network equilibrium model with hierarchical structures which, until now, was not formulated in an integrated way as a variational inequality formulation. This formulation permits the rigorous mathematical analysis of the model structure and the development of an efficient solution algorithm.

The remainder of the paper is organized as follows. In the next section, we provide a literature review of the related problems. In section 3, we introduce the notations and develop the variational inequality formulation. Section 4 provides the analysis of the mathematical structure of the model by using the corresponding KKT conditions, and the solution algorithm is given in section 5. In section 6, we provide computational results, and some conclusions are presented in the last section.

2. A Brief Literature Review

The development of network equilibrium models that may be formulated as optimization problems originates with the seminal contribution of Beckmann, Winsten and McGuire (1956). The introduction of variable demand network equilibrium models for one mode, where the destination choice is determined by entropy type distribution models, can be found in the contributions of Florian, Ferland and Nguyen (1975), and Evans (1976).

This model, referred to as combined trip distribution and assignment models, was extended to two modes and mode choice by Florian and Nguyen (1978) when the travel times by two modes, say auto and transit, are not related. These models are convex cost multi-commodity network optimization models that may be solved by the linear approximation method (Frank and Wolfe, 1956) or by the partial linear approximation method first suggested by Evans (1976). Florian (1977) formulated a two-mode network equilibrium model where the transit travel times depend on the auto travel times, and the auto travel times account for the pressure of the transit vehicles

which share the capacity of the roads.

This model was recognized to be a special instance of a variational inequality formulation following the work of Smith (1979) and Dafermos (1980) for network equilibrium models with asymmetric link cost functions. It was further analyzed by Fisk and Nguyen (1982) and Florian and Spiess (1982) and solved by adaptation of the nonlinear Jacobi method. Sufficient conditions for the convergence of the algorithm were developed by Dafermos (1982) and Pang and Chan (1982). Friesz (1980) developed an optimization formulation of the multi-class combined trip distribution, mode choice and assignment model where the travel costs depend on the mode chosen, and Fisk and Boyce (1983) developed a variational inequality formulation for this model where the travel costs are not mode interrelated and the mode choice is given by a multinomial logit function. In the attempt to introduce the trip generation model in an integrated model, Safwat and Magnanti (1988) developed an integrated model which has an equivalent convex cost optimization formulation, and Safwat, Nobil and Walton (1988) provide a calibration and computational experience with this model.

Lam and Huang (1992) present a multi-class combined trip distribution mode choice on assignment model formulated as well as an equivalent convex cost optimization problem and solve it with the partial linearization method. The first formulation of a network equilibrium model with hierarchical logit demand functions is that of Fernandez et al. (1994) where the choice of stations for the "park-n-ride" mode is given by a lower nest of the demand function. The model is formulated as an equivalent convex cost optimization model and the solution method suggested is an adaptation of the partial linear approximation algorithm. Oppenheim (1995) presents several versions of combined trip distribution mode choice and assignment models. The importance of properly stating and solving integrated network equilibrium models is stressed by Boyce (1998). A recent contribution is that of Abrahamsson and Lundqvist (1999) who developed a convex cost optimization model for the combined trip distribution, mode choice and assignment models with hierarchical choices, where distribution may precede mode choice or vice versa. In their model, the transit travel times are independent of the auto travel times.

The model developed in this paper is motivated by the ESTRAUS model developed for the city of Santiago, Chile and described in various internal technical reports. In this model, the trip productions are given by class and purpose of travel. The mode choice model is an aggregated hierarchical logit function (Ortuzar and Willumsen (1996, p. 219). There are thirteen classes of travelers, three travel purposes and multiple modes of travel, which include walk, auto, multiple transit modes and combined modes ("park-n-ride"). The model components were calibrated by using survey data, and a computational procedure was used for its numerical solution without the statement of an integrated model. In this paper, we formulate the model as a variational inequality formulation and show that the Karush-Kuhn-Tucker (KKT) conditions replicate all the model components. Then a solution algorithm based on a Block Gauss-Seidel decomposition of the model which is akin to the partial linear approximation method of Evans (1976) is used

to develop a solution algorithm.

3. Model Formulation

In order to present the model components and the variational inequality formulation, the following notation is used. The arcs of the road network are designated by $a, a \in A$, where A is the set of arcs. The segments of the transit lines which compare the transit networks are designated $b, b \in B$, where B is the set of segments of the transit network. $a(b)$ designated that segment b of a transit line segment uses link a of the road network. The demand for travel by user class $m, m \in M$, for purpose $p, p \in P$, class $n, n \in N$ and mode $m, m \in M$ for the origin-destination pair (i, j), is denoted T_{ij}^{pnm} where P, N, M are the sets of purposes, classes and modes. This demand may use paths $R_{ij}^{pnm} \in R$ where R is the set of all routes $R = \bigcup_{(i,j),p,n,m} R_{ij}^{pnm}$. It is also useful to identify M_1 as the subset of modes which use private vehicles (auto, taxi), M_2 as the subset of transit related modes and M_3 as the walk mode.

A hierarchical logit structure for each purpose has three levels: the root, the groups and the modes. G^p denotes a set of groups or nests of nodes for purpose p, and g denotes a set of modes in a group or nest. These are illustrated in Figure 1, where $G^p = \{g_1, g_2, g_3\}$, $g_1 = \{m_1\}$, $g_2 = \{m_2\}$ and $g_3 = \{m_3, m_4, m_5\}$

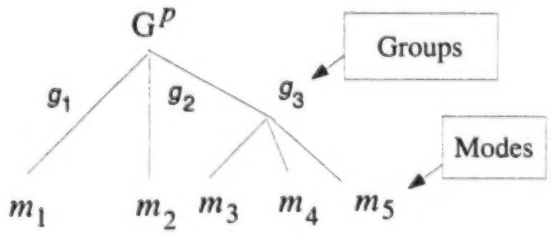

Figure 1. Nested logit structure for purpose p.

The inclusion coefficients of each group are given as $\phi_g^p = \begin{cases} 1 & if\ |g| = 1 \\ \phi_g^p & if\ otherwise \end{cases}$ all g, p. The production of trips are specified by origin i, purpose p and class n as O_i^{pn} while the attractions of trips are specified by destination j and purpose p, D_j^p. The travel demands T_{ij}^{pnm} give rise to path flows h_r^{pnm}, $r \in R_{ij}^{pnm}$. The feasible Ω region of the demands for travel and the path flows is stated next, where dual variables, which will be used later in the analysis are indicated in brackets.

$$\sum_g \sum_j T_{ij}^{png} = O_i^{pn}, \forall\, i, p, n \qquad (\alpha_i^{pn}) \qquad (1)$$

$$\sum_g \sum_n \sum_i T_{ij}^{png} = D_j^p, \forall\, j, p \qquad (\xi_i^p) \tag{2}$$

$$\sum_{m \in g} T_{ij}^{png} - T_{ij}^{png} = 0, \forall\, j, p, n, g \qquad (L_{ij}^{png}) \tag{3}$$

$$\phi_g^p \Big(\sum_{r \in R^m} h_r^{pnm} - T_{ij}^{pnm} \Big) = 0, \forall\, j, p, n.m \in g \qquad (\mu_{ij}^{pnm}) \tag{4}$$

$$h_r^{pnm} \geq 0, \ \ \forall\, r, p, n, m \qquad (\gamma_r^{pnm}) \tag{5}$$

$$T_{ij}^{pnm} > 0, \ \ \forall\, (i,j), p, n, m \tag{6}$$

$$T_{ij}^{png} > 0, \ \ \forall\, (i,j), p, n, g \tag{7}$$

Equations (1) and (2) are the balance equations for productions and attractions. Equation (3) relates mode totals to group totals. Equation (4) is the correction of flow equations for path flows. Note that the multiplication by ϕ_g^p is required for the analysis to follow. Equations (5), (6) and (7) are the usual positive and nonegativity requirements on path flows and demands respectively.

The presentation method that we have chosen is to state the variational inequality formulation first and then to show that the KKT conditions recover the model components. The variational inequality formulation supposes that β^{pn} is a calibrated parameter for all (p,n) and $\phi_g^p = (0,1)$ is the inclusion coefficient.

Find $(h^*, T^*) \in \Omega$ which satisfies

$$\sum_{pn} \sum_{(ij)} \Big[\sum_{g \subseteq G^p} \Big(\sum_{m \in g} \Big(\sum_r \phi_g^p C_r^{pnm}(h^*, T^*)(h_r - h_r^*) \Big) + \sum_{m \in g} \phi_g^p \ln(T_{ij}^{pnm^*} / T_{ij}^{png^*})(T_{ij}^{pnm} - T_{ij}^{pnm^*}) \Big)$$

$$+ \frac{1}{\beta^{pn}} \ln T_{ij}^{png^*} (T_{ij}^{png} - T_{ij}^{png^*}) \Big] \geq 0, \ \forall\, (h,T) \in \Omega. \tag{8}$$

It is worthwhile to note that (8) may be rewritten in the equivalent form:

$$\sum_{pn} \sum_{(ij)} \Big[\sum_{g \in G^p} \Big(\sum_{m \in g} \sum_r \phi_g^p C_r^{pnm}(h^*, T^*)(h_r - h_r^*) \Big) + \sum_{m \in g} \phi_g^p \ln(T_{ij}^{pnm^*})(T_{ij}^{pnm} - T_{ij}^{pnm^*}) \Big)$$

$$+ \Big(\frac{1}{\beta^{pn}} - \phi_g^p \Big) \ln T_{ij}^{png^*} (T_{ij}^{png} - T_{ij}^{png^*}) \Big] \geq 0, \ \forall\, (h,T) \in \Omega. \tag{9}$$

This formulation looks similar to some mathematical structures of combined trip distribution mode choice and assignment models which have equivalent convex cost optimization formula-

tions. See Fernandez et al. (1994) and Boyce (1998, pp. 81). Although the model presented here does not have an equivalent convex cost optimization formulation, the variational inequality approach provides a powerful way to analyze the model structure.

4. The Analysis of the Mathematical Structure of the Model

In this section, we derive and analyse the KKT conditions of the variational inequality formulation (35), which are given below.

$$T_{ij}^{png}: \quad \frac{1}{\beta^{pn}} \ln T_{ij}^{png} - \alpha_i^{pn} - \xi_i^p + L_{ij}^{png} = 0 \tag{10}$$

$$T_{ij}^{pnm}: \quad \phi_g^p \ln(T_{ij}^{pnm}/T_{ij}^{png}) - L_{ij}^{png} + \phi_g^p \mu_{ij}^{pnm} = 0 \tag{11}$$

$$h_r^{pnm}: \quad \phi_g^p C_r^{pnm} - \phi_g^p \mu_{ij}^{pnm} - \gamma_r^{pnm} = 0 \tag{12}$$

Complementarity:

$$h_r^{pnm} \gamma_r^{pnm} = 0 \tag{13}$$

$$\gamma_r^{pnm} \geq \ and \ (1) - (7)$$

From (10), we have

$$T_{ij}^{png} = e^{\beta^{pn}(\alpha_i^{pn} + \xi_j^p)} e^{-\beta^{pn} L_{ij}^{png}}. \tag{14}$$

From (11), we have

$$T_{ij}^{pnm}/T_{ij}^{png} = e^{(1/\phi_g^p) L_{ij}^{png}} e^{-\mu_{ij}^{pnm}}, m \in g. \tag{15}$$

Using (15) and (3), we have

$$e^{-(1/\phi_g^p) L_{ij}^{png}} = \sum_{m' \in g} e^{-\mu_{ij}^{pnm'}} \tag{16}$$

which implies

$$L_{ij}^{png} = -\phi_g^p \ln(\sum_{m' \in g} e^{-\mu_{ij}^{pnm'}}) \tag{17}$$

From (14) and (15), it follows that

$$T_{ij}^{pnm} = e^{\beta^{pn}(\alpha_i^{pn} + \xi_j^p)} e^{(1/\phi_g^p - \beta^{pn}) L_{ij}^{png}} e^{-\mu_{ij}^{pnm}}. \tag{18}$$

The analysis of (10) - (18) leads to the following propositions which emphasize the fact that the variational inequality formulation (8) implies both group and mode proportions ($P_{ij}^{png}, P_{ij}^{pnm}$) of demands computed by the hierarchical logit structure as well as the network equilibrium path choice conditions.

Proposition 1 (Group and Mode Proportions)

$$P_{ij}^{png} = \frac{T_{ij}^{png}}{T_{ij}^{pn}} = \frac{\epsilon^{-\beta^{pn} L_{ij}^{png}}}{\sum_{g' \subseteq G^p} \epsilon^{-\beta^{pn} L_{ij}^{png'}}} = \frac{\epsilon^{-\beta^{pn} L_{ij}^{png}}}{\epsilon^{-\beta^{pn} L_{ij}^{pn}}}, \quad g \in G^p \tag{19}$$

and

$$P_{ij}^{pnm} = \frac{T_{ij}^{pnm}}{T_{ij}^{png}} = \frac{\epsilon^{-u_{ij}^{pnm}}}{\sum_{m' \in g} \epsilon^{-u_{ij}^{pnm'}}} = \frac{\epsilon^{-u_{ij}^{pnm}}}{\epsilon^{-(1/\phi_g^p) L_{ij}^{png}}}, \quad m \in M. \tag{20}$$

Proposition 2 (Multinomial Logit Model)

If $\phi_g^p = 1 (\forall g \in G^p)$, then we obtain the traditional multinomial logit structure for purpose p, i.e.,

$$P_{ij}^{pnm} = \frac{T_{ij}^{pnm}}{T_{ij}^{pn}} = \frac{\epsilon^{-u_{ij}^{pnm}}}{\sum_{m' \in M} \epsilon^{-u_{ij}^{pnm'}}}, \quad m \in M.$$

Proposition 3 (Multi-Class Multi-Mode Equilibrium Conditions)

$$C_r^{pnm} \begin{cases} = \mu_{ij}^{pnm} & \text{if} \quad h_r^{pnm} > 0 \\ \geq \mu_{ij}^{pnm} & \text{if} \quad h_r^{pnm} = 0 \end{cases}, \forall r, p, n, m \in M$$

Proof: Conditions (12) and (13) imply the following equilibrium conditions.

The KKT conditions are also used for the algorithm development. It is important to recall the way that costs are propagated upward in the hierarchical logit demand structure. We define $L_{ij}^{png} = -\phi_g^p \ln(\sum_{m' \in g} \epsilon^{-\mu_{ij}^{pnm'}}), \forall (i,j), p, n, g$ as group logsum for purpose p, class n and group g (this definition is particular to the ESTRAUS model which inspired this model), where ϕ_g^p is the inclusion coefficient of group g (see Ortuzar and Willumsen, 1996, pp. 219).

We define $L_{ij}^{pn} = -\frac{1}{\beta^{pn}} \ln \sum_{g \subseteq G^p} \epsilon^{-\beta^{pn} L_{ij}^{png}}, \forall (i,j), p, n$ as top logsum for purpose p and class n. The total demands for each p, n may be obtained as the solution of the following multi-

proportional matrix balancing problem (Combined trip distribution/mode choice model), by noting that $T_{ij}^{pn} = \sum_{g \subseteq G^p} T_{ij}^{png}$,

$$T_{ij}^{pn} = e^{\beta^{pn}(\alpha_i^{pn}+\xi_j^p)} e^{-\beta^{pn} L_{ij}^{pn}} \tag{21}$$

$$\sum_j T_{ij}^{pn} = O_i^{pn}, \ \forall\, i, p, n \tag{22}$$

$$\sum_n \sum_i T_{ij}^{pn} = D_j^p, \ \forall\, j, p \tag{23}$$

$$T_{ij}^{pn} > 0, \ \forall ij, p, n \tag{24}$$

which may be easily shown to be equivalent to the following entropy maximization problem:

$$\begin{aligned} \max \ & -\sum_{pn} \sum_w \frac{1}{\beta^{pn}} T_w^{pn} (\ln T_w^{pn} - 1) \\ \text{s.t.} \quad & \sum_j T_{ij}^{pn} = O_i^{pn}, \forall\, i, p, n \\ & \sum_n \sum_i T_{ij}^{pn} = D_j^p, \forall j, p \\ & \sum_i \sum_j T_{ij}^{pn} L_{ij}^{pn} = \overline{L}^{pn}, \forall\, p, n \\ & T_{ij}^{np} > 0, \ \forall\, (i,j), p, n \end{aligned} \tag{25}$$

where $\overline{L}^{pn}$ is a constant (unknown) such as total generalized cost, and the dual variable associated with (25) is set to be 1. (This is exactly the trip distribution component of the ESTRAUS model).

5. A Solution Algorithm

In this section, we develop the solution algorithm used to obtain the solution of the variational inequality (9). We employ a Block Gauss-Seidel method (Equilibration Algorithm) where the network equilibrium formulation for given demands are computed in one block and the trip distribution/mode choice computations are computed in the other. The demand matrices for trips that use the vehicle network are computed at every successive iteration by using the method of successive averages over the vehicle flows. The solution algorithm flow chart is shown in the block diagram of Figure 2.

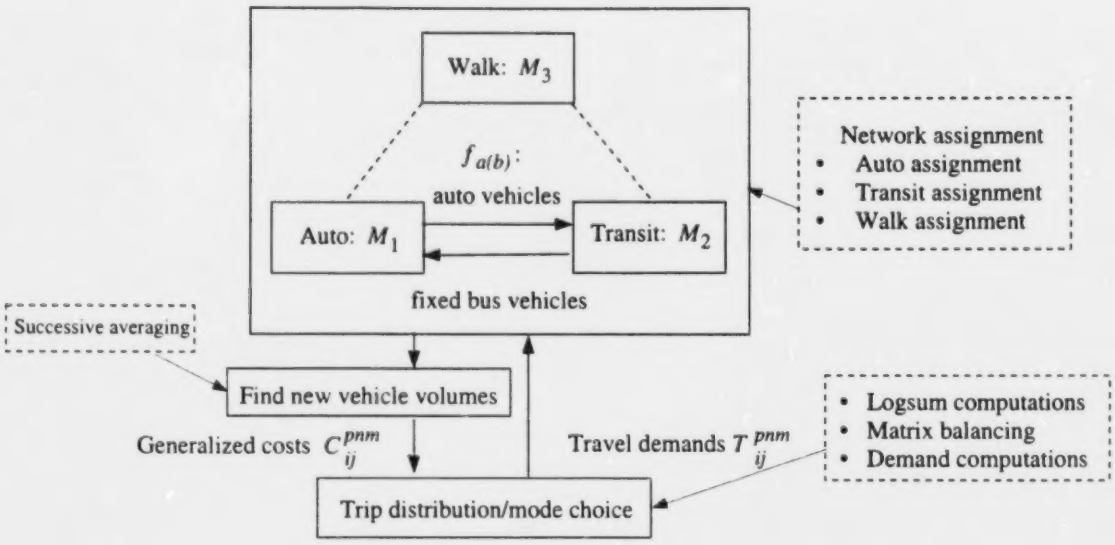

Figure 2. Block diagram of Equilibration Algorithm.

The network block requires the solution of the variational inequality formulation subject to constraints (27) to (34) as seen below. Since there are three distinct network modes M_1, M_2, M_3, one obtains three corresponding variational inequality problems which have equivalent optimization problems via a diagonalization. The network block is hence given by

$$\sum_{pn}\sum_{m\in M}\sum_{ij}\sum_{r\in R_{ij}^{pnm}} C_r^{pnm}(h^*,T)(h_r - h_r^*) \geq 0, \forall\, h \in \Omega(T). \tag{26}$$

$$s.t. \qquad \sum_r h_r^{pmn} = T_{ij}^{pnm}, \ \forall\,(ij), p, n, m \tag{27}$$

$$h_r^{pnm} \geq 0, \ \forall\, p, n, m \in M, r \in R_{ij}^{pnm}, (ij) \in W \tag{28}$$

where

$$f_a^1 = \sum_{pn}\sum_{m\in M_1}\sum_{r\in R_1} h_r^{pnm}\delta_{ar}, \quad \forall\, a \in A \tag{29}$$

$$f_b^2 = \sum_{pn}\sum_{m\in M_2}\sum_{r\in R_2} h_r^{pnm}\delta_{br}, \quad \forall\, b \in B \tag{30}$$

$$f_a^3 = \sum_{p}\sum_{m\in M_3}\sum_{r\in R_3} h_r^{pnm}\delta_{ar}, \quad \forall\, a \in A \tag{31}$$

$$C_r^{pnm} = \sum_{a \in A} c_a^{pnm}(f_a^1)\delta_{ar}, \quad \forall\, p,n,m \in M_1, r \in R_{ij}^{pnm} \tag{32}$$

$$C_r^{pnm} = \sum_{b \in B} c_b^{pnm}(f_{a(b)}^1)\delta_{br}, \quad \forall\, p,n,m \in M_2, r \in R_{ij}^{pnm} \tag{33}$$

$$C_r^{pnm} = \sum_{a \in A} c_a^{pnm}\delta_{ar}, \quad \forall\, p,n,m \in M_3, r \in R_{ij}^{pnm} \tag{34}$$

The use of the nonlinear Jacobi method permits to solve the three distinct network modes in sequence. First, the auto network problem (35) - (39) is solved as a fixed demand network equilibrium model for the network M_1.

$$\sum_{pn}\sum_{ij}\sum_{m \in M_1}\sum_{r} C_r^{pnm}(h^*,T)(h_r - h_r^*) \geq 0, \forall\, h \in \Omega(T). \tag{35}$$

$$s.t. \qquad \sum_{r} h_r^{pnm} = T_{ij}^{pnm}, \ \forall\,(ij), p,n,m \in M_1 \tag{36}$$

$$h_r^{pnm} \geq 0, \ \forall\, p,n,m \in M_1, r \in R_{ij}^{pnm} \tag{37}$$

where

$$f_a^1 = \sum_{pn}\sum_{m \in M_1}\sum_{r \in R_1} h_r^{pnm}\delta_{ar}, \quad \forall\, a \in A \tag{38}$$

$$C_r^{pnm} = \sum_{a \in A} c_a^{pnm}(f_a^1)\delta_{ar}, \quad \forall\, p,n,m \in M_1, r \in R_{ij}^{pnm} \tag{39}$$

Thus, the transit link costs can be determined to compute the transit flows (40) - (44) with link costs $c_b^{pnm}(f_{a(b)}^1)$ to compute the corresponding origin-destination costs.

$$\sum_{pn}\sum_{ij}\sum_{m \in M_2}\sum_{r} C_r^{pnm}(h^*,T)(h_r - h_r^*) \geq 0, \forall\, h \in \Omega(T). \tag{40}$$

$$s.t. \qquad \sum_{r} h_r^{pnm} = T_{ij}^{pnm}, \ \forall\,(ij), p,n,m \in M_2 \tag{41}$$

$$h_r^{pnm} \geq 0, \ \forall\, p,n,m \in M_2, r \tag{42}$$

where

$$f_b^2 = \sum_{pn}\sum_{m \in M_2}\sum_{r \in R_2} h_r^{pnm}\delta_{br}, \quad \forall\, b \in B \tag{43}$$

$$C_r^{pnm} = \sum_{b \in B} c_b^{pnm}(f_a^1)\delta_{br}, \quad \forall\, p,n,m \in M_2, r \in R_{ij}^{pnm} \tag{44}$$

Finally, the walk network model may be solved by an "all-or-nothing' assignment as the solution of (45) to (49).

$$\sum_{pn}\sum_{ij}\sum_{m \in M_3}\sum_{r} C_r^{pnm}(h^*,T)(h_r - h_r^*) \geq 0, \forall\, h \in \Omega(T). \tag{45}$$

$$s.t. \qquad \sum_r h_r^{pnm} = T_{ij}^{pnm}, \ \forall\,(ij), p,n,m \in M_3 \tag{46}$$

$$h_r^{pnm} \geq 0, \ \forall\, p,n,m \in M_3, r \in R_{ij}^{pnm} \tag{47}$$

where

$$f_a^3 = \sum_p \sum_{m \in M_3} \sum_{r \in R_3} h_r^{pnm}\delta_{ar}, \quad \forall\, a \in A \tag{48}$$

$$C_r^{pnm} = \sum_{a \in A} c_a^{pnm}\delta_{ar}, \quad \forall\, p,n,m \in M_3, r \in R_{ij}^{pnm} \tag{49}$$

The demand block requires the computation of the multi-proportional trip distribution model described in the previous sections, which is solved by an adaptation of the two-dimensional balancing method.

If $\phi_g^p = 1(\forall g \in G^p)$ and there are no constraints, (2), then (18) can be expresses as

$$T_{ij}^{pnm} = e^{\beta^{pn}\alpha_i^{pn}} e^{-u_{ij}^{pnm}}. \tag{50}$$

Using (1), (3) and (45), we obtain

$$e^{\beta^{pn}\alpha_i^{pn}} = O_i^{pn} / \sum_g \sum_m \sum_j e^{-\mu_{ij}^{pnm}} \tag{51}$$

With (27) and (51), we have

$$T_{ij}^{pnm} = O_i^{pn} e^{-u_{ij}^{pnm}} / \sum_g \sum_{m' \in g} \sum_j e^{-u_{ij}^{pnm'}}. \tag{52}$$

We call this case a "single constraint". The logsum costs L_{ij}^{pn} are computed by straightforward matrix computations according to the formulae stated in the previous section. The Equilibration

Algorithm which employs the nonlinear Jacobi method for the network blocks and the method of successive averages to dampen the demand is summarized below.

Equilibration Algorithm

Step 0 (Initialization): $l = 0$.
Choose on initial demand $\overline{T}^0$ (which equals the sum of auto and taxi trips) and compute $f^{1,0}$

Step 1 (Computation of generalized costs): $l = l + 1$,
Compute generalized costs $C^{pnm,l}, \forall(p,n,m)$ based on the current $f_a^{1,l}, c_a(f_a^{1,l})$ and computed transit costs $c_b(f_{a(b)}^{1,l})$.
Compute logsum matrices L_{ij}^{png} and $L_{ij}^{pn}, \forall(i,j),(p,n,g)$.

Step 2 (Combined trip distribution/mode choice model):
Carry out multi-proportional balancing (21)-(24) to obtain $T_{ij}^{pn}, \forall(p,n)$ and
Compute the O-D matrix of auto drivers $T^{pnm,l}\forall(p,n,m)$ (using the dual variables from the trip distribution model).

Step 3 (Vehicle network equilibrium assignment):
Compute (35)-(39) for $\overline{f}^{1,l}$ with $T_{ij}^{pnm}\forall(i,j)(p,n,m \in M_1)$

Step 4 (Convergence test):
If $\|f^{1,l} - \overline{f}^{1,l}\| \leq \varepsilon(\varepsilon > 0)$, go to Step 6.

Step 5 (Method of Successive Averages):
Compute $f^{1,l+1} = f^{1,l}(x) + \overline{f}^{1,l}(1-x), (x = 1/(l+1))$ and return to Step 1.

Step 6 (Computation of final results): compute $T_{ij}^{pnm}, \forall(i,j), p, n, m$
Compute final f^1, f^2, f^3.
Compute final $C_{ij}^{pnm}, \forall(i,j), p, n, m$.

6. Computational results (STGO Model)

This model is implemented with EMME/2 and is applied in Santiago, Chile. The model is called STGO. And while there are many details, such as parameters and equations (required for computation), it would server no purpose to discuss them here. However, we provide the following basic information to specify the STGO model. See Table 1 for the STGO base network specification, Figure 3 for the network and Table 2 for the travel modes.

Table 1. STGO base network specifications

Supply Modes		12	Regular nodes	1091
Travel Modes		11	Directional links	5606
Centroids	Demands zones	264	Transit lines	1273
	Parking zones	47	Transit line segments	45753
Classes		13	Trip purposes	3

Table 2. Travel modes

Travel mode	Description	Travel mode	Description
1 (wk)	Walk	7 (mt)	Metro
2 (ad)	Auto-driver	8 (bm)	Bus-metro
3 (ap)	Auto-passenger	9 (dm)	Auto-driver-metro
4 (tx)	Taxi	10 (pm)	Auto-passenger-metro
5 (tc)	Taxi-col	11 (tm)	Taxi-col-metro
6 (bs)	Bus		

Figure 3. Santiago Network

We note that different groups (nests) of travel modes are established according to trip purposes, with different parameters. Here the EMME/2 implementation on a PC can effectively deal with the combined modes, such as park and ride (see modes 9 and 10).

The implementation has the following features. Transit in-vehicle times depend on the vehicle travel time in the vehicle network via transit functions. The generalized costs for transit related travel modes depend on the fares, mode specific walk times, in-vehicle times, waiting times and boarding times. The generalized costs for vehicle related travel modes depend on vehicle time and distances. The interactions between M_1, M_2, M_3 are given in Figure 4 where the dash lines imply no interactions implemented and the solid lines mean that there are interactions. In Step 1, the transit related generalized costs are computed as done in the ESTRAUS model where five transit assignments are performed with each for one transit mode. The corresponding computed components (such as in-vehicle times and fares) are used to compute the generalized costs. In Step 3, the vehicle network equilibrium assignment (35)-(39) is performed as done in the ESTRAUS model with a set of calibrated volume delayed functions. On a PC (PII 450, 96M), it took 20 min per iteration. About ten iterations are needed to obtain very similar results to that of ESTRAUS, if the initial demand is zero. If we want to perform a full run for a new scenario, we may only need three to five iterations.

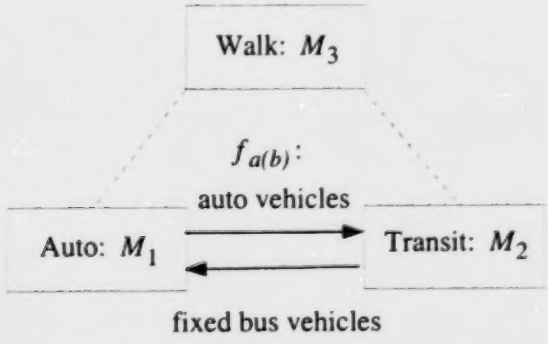

Figure 4. Interactions among the three assignment components

The convergence of the Equilibration Algorithm for the STGO model is reported in Table 3. We see that after about ten iterations, the total demand stabilizes. Figure 5 shows very similar demand totals by mode computed by both the ESTRAUS and STGO models.

Table 3. Convergence of the procedure (30 loops).

	DEMAND		LINK VOLUME RELATIVE DIFFERENCE		
LOOPS	demand	Average demand relative difference (percentage)	Total	max.	average
1	96.720695	0	3616	1	1
2	329579.5	94.80645	3962.34545	1	0.999583
3	321322	49.46157	1853.61218	55.236835	0.46832
...	...	...	...	...	...
9	294608.687	1.113139	874.864257	99.95684	0.222555
10	294363,687	0.832489	830.680847	89.338211	0.211154
11	294199.062	0.627432	782.620056	95.591217	0.198988
12	294104	0.53044	644.677001	77.937889	0.163957
...	...	...	...	...	...
30	293717.406	0.078193	333.711791	27.862077	0.084849
31	293736.562	0.05948	307.420928	28.45512	0.078164

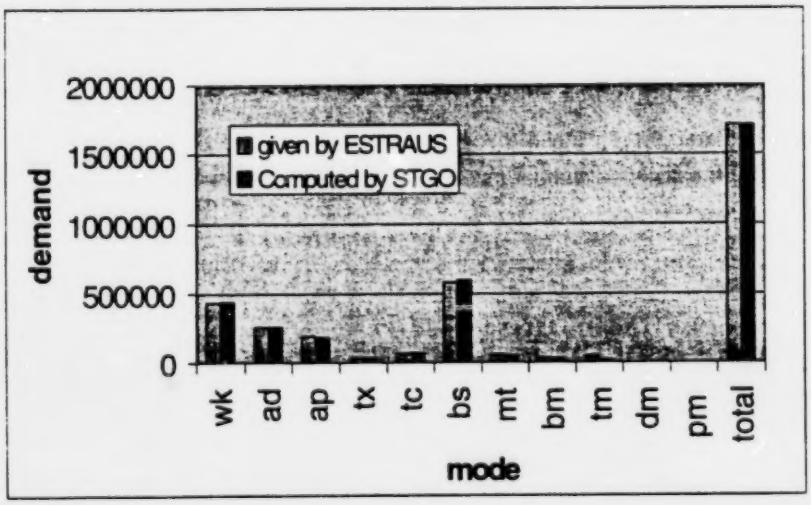

Figure 5. Demand totals for each mode between STGO and ESTRAUS

The advantages of using the variational inequality formulation are that it is more flexible and general by comparison with the limited optimization formulation. Based on this formulation,

we derived a Block Gauss-Seidel method. We have not seen any optimization formulations developed so far which can result in the corresponding equations being used in practice.

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